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JHEP05(2009)051

(http://iopscience.iop.org/1126-6708/2009/05/051)

Published by IOP Publishing for SISSA

Received: *March 22, 2009*
Revised: *April 29, 2009*
Accepted: *May 2, 2009*
Published: *May 14, 2009*

Shear viscosity, instability and the upper bound of the Gauss-Bonnet coupling constant

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Abstract: We compute the dimensionality dependence of η/s for charged black branes with Gauss-Bonnet correction. We find that both causality and stability constrain the value of Gauss-Bonnet coupling constant to be bound by 1/4 in the infinite dimensionality limit. We further show that higher dimensionality stabilize the gravitational perturbation. The stabilization of the perturbation in higher dimensional space-time is a straightforward consequence of the Gauss-Bonnet coupling constant bound.

Keywords: Duality in Gauge Field Theories, AdS-CFT Correspondence

doi:10.1088/1126-6708/2009/05/051

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Contents

1 Introduction

The AdS/CFT correspondence [1–3] provides an interesting theoretical framework for studying relativistic hydrodynamics of strongly coupled gauge theories. The result of RHIC experiment on the viscosity/entropy ratio turns out to be in favor of the prediction of AdS/CFT [4–6]. Some attempt has been made to map the entire process of RHIC experiment in terms of gravity dual [7]. The way to include chemical potential in the theory was figured out in [8, 9]. Phases of these theories were also discussed in [9–13].

It had been conjectured that the viscosity value of theories with gravity dual may give a lower bound for the $\eta/s = \frac{1}{4\pi}$ for all possible liquid [14]. However, in the presence of higher-derivative gravity corrections, the viscosity bound and causality are also violated as a consequence [15–22].

The higher derivative gravity terms are related to the (in)stability issues of black holes. The black hole stability issues are a crucial problem because black hole solutions are no longer unique in spacetime with higher than four dimensions. The instability of D-dimensional asymptotically flat Einstein-Gauss-Bonnet black holes has been discussed by several authors [23, 24]. Their results show that for the gravitational perturbations of Schwarzschild black holes in $D \geq 5$ Gauss-Bonnet gravity, the instability occurs only for $D = 5$ and $D = 6$ cases at large value of α' [24]. In the previous paper [20], we computed the charge dependence of η/s for Gauss-Bonnet theory and noticed that charges introduced instability of the black brane even in the range $0 < \lambda \leq 0.09$.

The purpose of this paper is to perform a complete computation of η/s including the charge and Gauss-Bonnet correction to an arbitrary dimensionality , and to determine the causality and stability constraints on the parameters of the black hole. Both the causality and stability constraints give the same result that λ should be bounded by $1/4$ for an arbitrary high dimensionality. We further find that higher dimensionality stabilize the tensor type perturbation.

2 Viscosity to entropy density ratio

We have explored the charge dependence of η/s in the presence of Gauss-Bonnet term for five-dimensional AdS black branes [20]. In this section, we generalize the previous result on η/s [20] to D-dimensional cases. Let us start by introducing the following action in D dimensions which includes Gauss-Bonnet terms and U(1) gauge field:

$$I = \frac{1}{16\pi G_D}\int \mathrm{d}^D x\sqrt{-g}\Big(R - 2\Lambda + \alpha' \left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) - 4\pi G_D F_{\mu\nu}F^{\mu\nu}\Big), \tag{2.1}$$

where α' is a (positive) Gauss-Bonnet coupling constant with dimension $(\text{length})^2$ and the field strength is defied as $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$. The thermodynamics and geometric properties of black objects in Gauss-Bonnet gravity were studied in several papers [25–31].

The charged black brane solution in D dimensions for this action is described by [28]

$$\mathrm{d}s^2 = -H(r)N^2\mathrm{d}t^2 + H^{-1}(r)\mathrm{d}r^2 + \frac{r^2}{l^2}\mathrm{d}x^i\mathrm{d}x^j, \tag{2.2a}$$

$$A_t = -\frac{Q}{4\pi(D-3)r^{D-3}}, \tag{2.2b}$$

with

$$\begin{aligned} H(r) &= \frac{r^2}{2\alpha}\left[1 - \sqrt{1 - \frac{4\alpha}{l^2}\left(1 - \frac{ml^2}{r^{D-1}} + \frac{q^2l^2}{r^{2D-4}}\right)}\,\right], \\ &= \frac{r^2}{2\lambda l^2}\left[1 - \sqrt{1 - 4\lambda\left(1 - \frac{r_+^{D-1}}{r^{D-1}} - a\frac{r_+^{D-1}}{r^{D-1}} + a\frac{r_+^{2D-4}}{r^{2D-4}}\right)}\,\right], \\ \Lambda &= -\frac{(D-1)(D-2)}{2l^2}. \end{aligned} \tag{2.3}$$

where α and α' are connected by a relation $\alpha = (D-4)(D-3)\alpha'$, $\lambda = \alpha/l^2$, $a = \frac{q^2l^2}{r_+^{2D-4}}$ and the parameter l corresponds to AdS radius. The horizon is located at $r = r_+$. The gravitational mass M and the charge Q are expressed as

$$\begin{aligned} M &= \frac{(D-2)V_{D-2}}{16\pi G_D}m, \\ Q^2 &= \frac{2\pi(D-2)(D-3)}{G_D}q^2. \end{aligned}$$

Taking the limit $\alpha' \to 0$, the solution corresponds to one for Reissner-Nordström-AdS (RN-AdS). The hydrodynamic analysis in this background has been done in [32, 33].

The constant N^2 in the metric (2.2a) can be fixed at the boundary whose geometry would reduce to flat Minkowski metric conformaly, i.e. $\mathrm{d}s^2 \propto -c^2\mathrm{d}t^2 + \mathrm{d}\vec{x}^2$. On the boundary $r \to \infty$, we have

$$H(r)N^2 \to \frac{r^2}{l^2},$$

so that N^2 is found to be

$$N^2 = \frac{1}{2}\left(1 + \sqrt{1 - 4\lambda}\,\right). \tag{2.4}$$

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Note that the boundary speed of light is specified to be unity $c = 1$. Eq. (2.4) implies that the significant value of λ lies in the region $\lambda \leq 1/4$. We will confirm this result from the causality and stability analysis in section 3 and 4.

The temperature at the event horizon is defined as

$$T = \frac{1}{2\pi\sqrt{g_{rr}}}\frac{\mathrm{d}\sqrt{g_{tt}}}{\mathrm{d}r} = \frac{Nr_+}{4\pi l^2}\left((D-1)-(D-3)a\right). \tag{2.5}$$

The black brane approaches extremal when $a \to \frac{D-1}{D-3}$ (i.e. $T \to 0$). The entropy density is given by [27]

$$s = \frac{1}{4G_D}\frac{r_+^{D-2}}{l^{D-2}}. \tag{2.6}$$

We will calculate the shear viscosity of the boundary theory using the Kubo formula

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dtd\vec{x}\mathrm{e}^{-i\omega t} < [T_{xy}(x), T_{xy}(0)] >= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}G_{xy,xy}(\omega,0), \tag{2.7}$$

where $G(\omega, 0)$ is the retarded Greens function for T_{xy}:

$$G_{xy,xy}(\omega,k) = -i\int dtdx\mathrm{e}^{ik\cdot x+i\omega t}\theta(t) < [T_{xy}(x), T_{xy}(0)] > \tag{2.8}$$

It is convenient to introduce coordinate in the following computation

$$z = \frac{r}{r_+}, \qquad \omega = \frac{l^2}{r_+}\bar{\omega}, \qquad k_3 = \frac{l^2}{r_+^2}\bar{k}_3, \qquad f(z) = \frac{l^2}{r_+^2}H(r),$$
$$f(z) = \frac{z^2}{2\lambda}\left[1-\sqrt{1-4\lambda\left(1-\frac{a+1}{z^{D-1}}+\frac{a}{z^{2D-4}}\right)}\right] \tag{2.9}$$

We now study the tensor type perturbation $h_y^x(t, x_3, z) = \phi(t, x_3, z)$ on the black brane background of the form

$$ds^2 = -f(z)N^2\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{b^2 f(z)} + \frac{z^2}{b^2 l^2}\left(2\phi(t,x_3,z)\mathrm{d}x\mathrm{d}y + \sum_{i=1}^{D-2}\mathrm{d}x_i^2\right), \tag{2.10}$$

where $b = \frac{1}{r_+^2}$. Using Fourier decomposition

$$\phi(t,x_3,z) = \int\frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}}\mathrm{e}^{-i\bar{\omega}t+i\bar{k}_3x_3}\phi(k,z),$$

we can obtain the following linearized equation of motion for $\phi(z)$ from the Einstein-Gauss-Bonnet-Maxwell field equation:

$$g(z)\phi'' + g'(z)\phi' + g_2\phi = 0 \tag{2.11}$$

where

$$g(z) = z^{D-2}f\left\{1-\frac{2\lambda}{D-3}\left[z^{-1}f' + z^{-2}(D-5)f\right]\right\} \tag{2.12}$$
$$g_2 = g(z)\frac{\omega^2}{N^2f^2} - k_3^2z^{D-4}\times$$
$$\left[1-\frac{2\lambda}{(D-3)(D-4)}\left(f'' + (D-5)(D-6)z^{-2}f + 2(D-5)z^{-1}f'\right)\right],$$

and the prime denotes the derivative with respect to z.

For the convenient calculation of the shear viscosity, we further introduce a new variable $u(=\frac{1}{z})$. Then we solve the equation of motion for transverse graviton eq. (2.11) in hydrodynamic regime i.e. small ω and k. The solution to eq. (2.11) as

$$\phi(z) = (1-u)^{\nu} F(u), \tag{2.13}$$

where $F(u)$ is a regular function at the horizon $u = 1$, so that the singularity at the horizon might be extracted. The parameter ν can be fixed as $\nu = \pm i\omega/4\pi T$ by substituting eq. (2.13) into the equation of motion. Usually we choose

$$\nu = -i\frac{\omega}{4\pi T} \tag{2.14}$$

as the incoming wave condition. To obtain the shear viscosity via Kubo formula (2.7), we only need know the $\omega \to 0$ behavior of the transverse graviton, so it is sufficient to expand $F(u)$ in terms of frequencies up to the linear order of $\omega(= i4\pi T\nu)$,

$$F(u) = F_0(u) + \nu F_1(u) + \mathcal{O}(\nu^2, k^2). \tag{2.15}$$

Expanding (2.11) to the first order of ν, we get the following form,

$$\left[g(u)F'(u)\right]' - \nu\left(\frac{1}{1-u}g(u)\right)' F(u) - \frac{2\nu}{1-u}g(u)F'(u) = 0. \tag{2.16}$$

Substituting the series expansion (2.15) into the equation (2.16), one can get the equations of motion for $F_0(u)$ and $F_1(u)$ recursively. Following the procedure given in [20], we easily get

$$F_0(u) = C, \qquad \text{(const.)}. \tag{2.17}$$

and

$$F_1'(u) = \frac{C}{1-u} + \frac{C_2}{g(u)}. \tag{2.18}$$

The integration constant C_2 can be fixed by the regularity condition of $F_1(u)$ at the horizon. So, the regularity condition at $u = 1$ implies

$$C_2 = -\left[((D-1)-(D-3)a)\left(1 - \frac{2\lambda}{D-3}((D-1)-(D-3)a)\right)\right]C. \tag{2.19}$$

The remaining constant C is estimated in terms of boundary value of the field,

$$\lim_{u\to 0}\phi(z) = \phi^{(0)},$$

so that we could fix

$$C = \phi^{(0)}\Big(1 + \mathcal{O}(\nu)\Big). \tag{2.20}$$

Now let us calculate the retarded Green function. Using the equation of motion, the action reduces to the surface terms. The relevant part is given as

$$I[\phi(u)] = -\frac{r_+^{D-1}N}{16\pi G_D l^D}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\Big(g(u)\phi(u)\phi'(u) + \cdots\Big)\Bigg|_{u=0}^{u=1}. \tag{2.21}$$

Near the boundary $u = \varepsilon$, using the obtained perturbative solution for $\phi(u)$, we can get

$$\begin{aligned}\phi'(\varepsilon) &= -\nu \frac{\left[((D-1)-(D-3)a)\left(1-\frac{2\lambda}{D-3}((D-1)-(D-3)a)\right)\right]}{g(\varepsilon)}\phi^{(0)} + \mathcal{O}(\nu^2, k^2) \\ &= i\omega\left(\frac{l^2}{4Nr_+}\right)\frac{1-\frac{2\lambda}{D-3}((D-1)-(D-3)a)}{g(\varepsilon)}\phi^{(0)} + \mathcal{O}(\omega^2, k^2). \end{aligned} \tag{2.22}$$

Therefore we can read off the correlation function from the relation (2.8),

$$G_{xy\ xy}(\omega, k) = -i\omega\frac{1}{16\pi G_D}\left(\frac{r_+^{D-2}}{l^{D-2}}\right)\left(1-\frac{2\lambda}{D-3}\left[(D-1)-(D-3)a\right]\right) + \mathcal{O}(\omega^2, k^2), \tag{2.23}$$

where we subtracted contact terms. Then finally, we can obtain the shear viscosity by using Kubo formula (2.7),

$$\eta = \frac{1}{16\pi G_D}\left(\frac{r_+^{D-2}}{l^{D-2}}\right)\left(1-\frac{2\lambda}{D-3}[(D-1)-(D-3)a]\right). \tag{2.24}$$

The ratio of the shear viscosity to the entropy density is found to be

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1-\frac{2\lambda}{D-3}[(D-1)-(D-3)a]\right). \tag{2.25}$$

The above result agrees with [20] when $D = 5$. Since λ is bounded by $1/4$, the shear viscosity never approaches zero in higher than $5D$ Gauss-Bonnet theory. When $a = 0$ (no charges), $\eta/s = (1 - \frac{2\lambda(D-1)}{(D-3)})/(4\pi)$, we recover the result in ref. [16]. It is also worth noting that for extremal case ($a = \frac{D-1}{D-3}$), the ratio of the shear viscosity to entropy density receives no corrections from Gauss-Bonnet terms. In the next two sections, we will show explicitly causality and stability impose more rigorous constraint on the value of λ and the upper bound of λ is $1/4$ as D approaches infinity.

3 Causality constraints

The authors in [16, 17] demonstrated that the causality could be violated if one introduced Gauss-Bonnet terms. In [20], it is found that the presence of charge does not contribute to causality and the result of [16, 17] is universal for charged black branes. In this section, we investigate the dimensionality dependence of the causality constraints.

Due to higher derivative terms in the gravity action, the equation (2.11) for the propagation of a transverse graviton differs from that of a minimally coupled massless scalar field propagating in the same background geometry. Writing the wave function as

$$\phi(x_3, z) = \mathrm{e}^{-i\omega t + ikz + ik_3 x_3}, \tag{3.1}$$

and taking large momenta limit $k^\mu \to \infty$, one can find that the equation of motion (2.11) reduces to

$$k^\mu k^\nu g_{\mu\nu}^{\text{eff}} \simeq 0, \tag{3.2}$$

where the effective metric is given by

$$\mathrm{d}s_{\mathrm{eff}}^2 = g_{\mu\nu}^{\mathrm{eff}}\mathrm{d}x^\mu \mathrm{d}x^\nu = N^2 f(r)\left(-\mathrm{d}t^2 + \frac{1}{c_g^2}\mathrm{d}x_3^2\right) + \frac{1}{f(r)}\mathrm{d}r^2. \tag{3.3}$$

Note that c_g^2 can be interpreted as the local speed of graviton:

$$c_g^2(z) = \frac{N^2 f}{z^2}\frac{1 - \frac{2\lambda}{(D-3)(D-4)}\left(f'' + (D-5)(D-6)z^{-2}f + 2(D-5)z^{-1}f'\right)}{1 - \frac{2\lambda}{D-3}\left[z^{-1}f' + z^{-2}(D-5)f^2\right]}. \tag{3.4}$$

We can expand c_g^2 near the boundary $z \to \infty$,

$$\begin{aligned} c_g^2 - 1 = &\left(-\frac{(D^2-5D+10)(1+a)}{2(D-3)(D-4)}\right. \\ &\left.+\frac{(D-1)(1+a)}{(D-3)(D-4)(1-4\lambda)} - \frac{1+a}{2\sqrt{1-4\lambda}}\right)\frac{1}{z^{D-1}} + \mathcal{O}(z^{-D}). \end{aligned} \tag{3.5}$$

As the local speed of graviton should be smaller than 1 (the local speed of light of the boundary CFT), we require

$$-\frac{(D^2-5D+10)}{2(D-3)(D-4)} + \frac{(D-1)}{(D-3)(D-4)(1-4\lambda)} - \frac{1}{2\sqrt{1-4\lambda}} \le 0. \tag{3.6}$$

The above formula leads to

$$\lambda \le \frac{D^4 - 10D^3 + 41D^2 - 92D + 96}{4(D^2-5D+10)^2}. \tag{3.7}$$

without any charge dependence. As D is large enough, the above formula becomes

$$\lim_{D\to\infty} \lambda_{\mathrm{causality}} \le \frac{1}{4}. \tag{3.8}$$

We can rewrite the above equation from the relation $\lambda = (D-3)(D-4)\alpha'/l^2$,

$$\frac{\alpha'}{l^2} \le \frac{D^4 - 10D^3 + 41D^2 - 92D + 96}{4(D^2-5D+10)^2(D-3)(D-4)}. \tag{3.9}$$

Figure 1 demonstrates that causality constrains the value of λ. When $D = 5$, it goes as $\lambda \le 0.09$ same as the result of [17] and when $D \to \infty$, λ is bounded by 1/4.

Now, we rewrite the wave function in a Schrödinger form,

$$-\frac{d^2\psi}{dr_*^2} + V\left(z(r_*)\right)\psi = \omega^2\psi, \qquad \frac{dr_*}{dz} = \frac{1}{Nf(z)}, \tag{3.10}$$

where $\psi\left(z(r_*)\right)$ and the potential is defined by

$$\begin{aligned} \psi &= K(z)\phi, \qquad K(z) \equiv \sqrt{\frac{g(z)}{z^{D-2}f(z)}}, \quad V = k^2 c_g^2 + V_1(z), \\ V_1(z) &\equiv N^2\left[\left(f(z)\frac{\partial \ln K(z)}{\partial z}\right)^2 + f(z)\frac{\partial}{\partial z}\left(f(z)\frac{\partial \ln K(z)}{\partial z}\right)\right] \end{aligned} \tag{3.11}$$

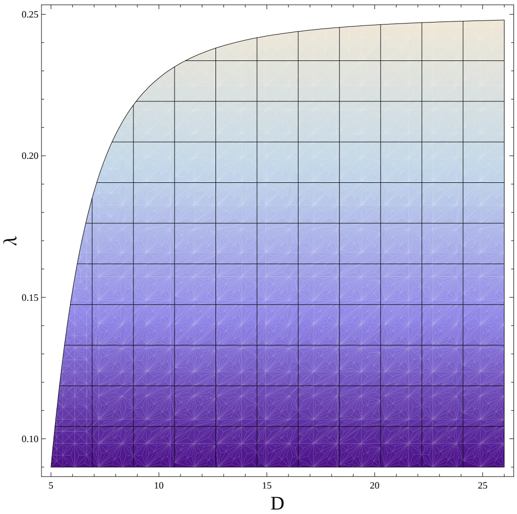

Figure 1. The minimal value of λ constrained by causality. The upper bound of λ is $1/4$.

From the geodesic equation of motion

$$g^{\text{eff}}_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s}=0, \tag{3.12}$$

and the Bohr-Sommerfield quantization condition

$$\int \mathrm{d}r_*\sqrt{\omega^2-k^2c_g^2}=\left(n-\frac{1}{4}\right)\pi, \tag{3.13}$$

one can find that the group velocity of the test particle along the geodesic line is given by [17]

$$v_g=\frac{\mathrm{d}\omega}{\mathrm{d}k}\to c_g>1. \tag{3.14}$$

Therefore, signals in the boundary CFT propagate outside of the light cone and micro-causality violation happens (for a more detailed and explicit discussion on causality violation, see [17]). Now we can conclude that as dimensions of space-time go up, causality restricts the value of λ in the region $\lambda \le 1/4$. In next section, we will prove that in the extremal limit $a \to \frac{D-1}{D-3}$, the stability of the black brane also requires that λ should also be bounded by $1/4$ in the limit $D\to\infty$.

4 Stability constraints

In [20], it was found that apart from the causality violation, for RN-AdS black brane in Gauss-Bonnet theory, the charges give new instability of the black brane within the window of $0<\lambda\le 0.09$. Now, we will show that higher D stabilize the gravitational perturbation.

From figure 2, we can see that the Schrödinger potential develops a negative gap near the horizon, but the gap becomes flatter as D increases. We will now show that while in the large momentum limit, the negative-valued potential leads to instability of the black

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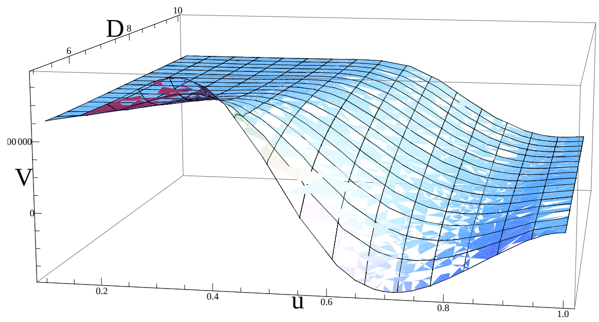

Figure 2. Schrodinger potential V(u) as a function of u ($u = \frac{1}{z}$) and D for $\lambda = 0.24$ and $a = \frac{D-1}{D-3}$.

brane, higher D tends to suppress those unstable perturbations. In the large momenta limit $k^\mu \to \infty$, the dominant contribution to the potential is given by $k^2 c_g^2$. In [20], it was found that for near extremal cases, c_g^2 can be negative near the horizon and $V \simeq k^2 c_g^2$ can be deep enough (see figure 2). Thus bound states can live in the negative-valued well. The negative energy bound state corresponds to modes of tachyonic mass on Minkowski slices [34] and signals an instability of the black brane [23, 24]. As D goes up, we will see new physics in the following. Let us expand c_g^2 in series of $(1 - \frac{1}{z})$,

$$\begin{aligned} c_g^2 = \; & [(D-3)a - D - 1]\left(1 + \sqrt{1-4\lambda}\right)\left\{D^2\left[4\lambda^2(a-1)^2 + 2(a+1)\lambda - 1\right]\right. \\ & \left. -D\left[8\lambda^2(3a^2-4a+1) + 2\lambda(a+7)\right] + \left[\lambda^2(3a-1)^2 - 3\lambda(a-1) - 3\right]\right\} \\ & \{2(D-4)\left[-3 + (2-6a)\lambda + D(1+2(a-1)\lambda)\right]\}^{-1}\left(1 - \frac{1}{z}\right) + \mathcal{O}\left(\left(1 - \frac{1}{z}\right)^2\right). \end{aligned} \tag{4.1}$$

Since $0 \le a \le \frac{D-3}{D-1}$, and $0 \le \frac{1}{z} \le 1$, c_g^2 will be negative, if

$$\begin{aligned} & \left\{D^2\left[4\lambda^2(a-1)^2 + 2(a+1)\lambda - 1\right]\right. \\ & \left. -D\left[8\lambda^2(3a^2-4a+1) + 2\lambda(a+7)\right] + \left[\lambda^2(3a-1)^2 - 3\lambda(a-1) - 3\right]\right\} \\ & \{2(-4+D)\left[-3 + (2-6a)\lambda + D(1+2(-1+a)\lambda)\right]\}^{-1} < 0. \end{aligned} \tag{4.2}$$

From the above formula, we find the critical value of λ,

$$\begin{aligned} \lambda_c(D,a) = \; & \frac{1}{4}\Bigg\{ -(D-1)(D-6) + (D-3)(D+2)a \\ & + \Bigg\{(D-1)^2(5D^2 - 40D + 84) + (D-3)^2(5D^2 - 24D + 52)a^2 \\ & -6a(D^2 - 8D + 20)(D-3)(D-1)\Bigg\}^{\frac{1}{2}}\Bigg\}\Bigg\{1 + (D-3)a - D\Bigg\}^{-2}. \end{aligned} \tag{4.3}$$

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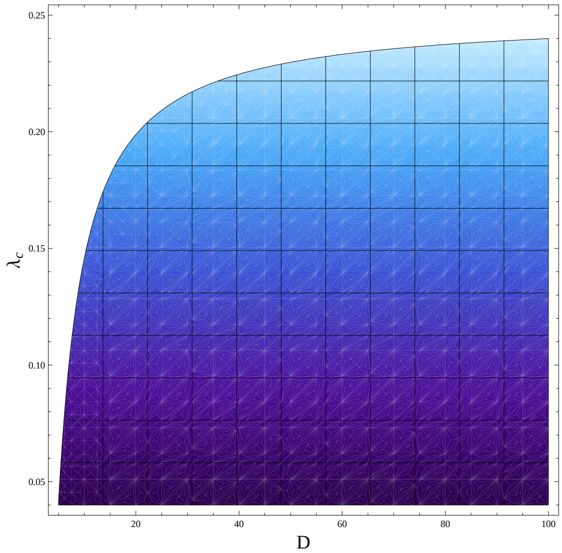

Figure 3. The minimal value of λ constrained by instability in the limit $a \to \frac{D-1}{D-3}$. The figure shows that λ_c is bounded by 0.25

Above the line of λ_c, c_g^2 can be negative (see figure 3). The minimal value of λ_c can be obtained in the limit $a \to (\frac{D-1}{D-3})$,

$$\lambda_{c,\ min} = \frac{1}{4}\frac{(D-3)(D-4)}{(D-1)(D-2)}. \tag{4.4}$$

That is to say

$$\left(\frac{\alpha'}{l^2}\right)_{c,\ min} = \frac{1}{4(D-1)(D-2)}. \tag{4.5}$$

When $D = 5$, $\lambda_{c,\ min} = \frac{1}{24}$, we recover the result obtained in ref. [20]. Eq. (4.4) indicates that for any value of a, the quasinormal modes (QNMs) become stable under the line $\lambda_{c,\ min}$.

As the value of D increases, one finds that $\lambda_{c,\ min}$ is also bounded by 1/4, i.e.

$$\lim_{(D,a)\to(\infty,\frac{D-1}{D-3})} \lambda_c = \frac{1}{4} \tag{4.6}$$

Note that this value is obtained in the extremal limit. Different from causality violation, the stability of the black brane depends on the charge. It would be very interesting to see for fixed value of charge, for which value of λ the QNMs become stable. One simple way is to go to concrete dimensionality (i.e. fix D first), then the relation between λ and a can be determined from eq. (4.3). Eq. (4.3) tells us that for $\lambda < \lambda_c(D,a)$, the black brane is always stable. Actually, as pointed out in [20], for fixed D the two lines $\lambda_c(a)$ and $\lambda_{causality}$ separates the physics into four regions in (a, λ) space: consistent region; only causality violation region; only unstable modes region; causality violation and unstable modes region(see figure 4 in [20] for more details). In order to show explicitly the behavior of gravitational perturbation in higher dimensions, we solve the Schrödinger equation (3.10)

D	$\lambda = 0.20$	$\lambda = 0.18$	$\lambda = 0.16$	$\lambda = 0.12$	$\lambda = 0.10$
6	$122.7276i$	$92.5503i$	$66.0012i$	$18.5027i$	–
7	$75.3435i$	$52.8819i$	$32.5866i$	–	–
8	$49.7368i$	$30.9208i$	16.8584	–	–
9	$34.7918i$	$20.0792i$	$7.5717i$	–	–
10	$24.7602i$	$11.9079i$	–	–	–

Table 1. Unstable QNMs for charged GB black brane perturbation of tensor type for fixed charge ($a = 1.20$) and $k_3 = 500$.

D	$a = 1.4$	$a = 1.2$	$a = 1.0$	$a = 0.8$	$a = 0.6$	$a = 0.4$
6	$129.001i$	$122.7276i$	$110.9159i$	$91.1455i$	$59.3517i$	$7.0564i$
7	$79.3218i$	$75.3435i$	$66.5505i$	$37.3177i$	$24.3269i$	–
8	$70.0214i$	$49.7368i$	$42.0717i$	$26.6617i$	–	–
9	–	$32.6603i$	$27.1475i$	$12.6057i$	–	–
10	–	$24.7602i$	$20.6077i$	$4.6077i$	–	–

Table 2. Unstable QNMs for charged GB black brane perturbation of tensor type for fixed λ ($\lambda = 0.20$) and $k_3 = 500$. Note that $a = 1.4$ exceeds the maximal value of charge permitted for 9- and 10-dimensional charged black brane and thus we leave the frequency blank there.

λ	$a = 1.6$	$a = 1.4$	$a = 1.2$	$a = 1.0$	$a = 0.8$	$a = 0.6$	$a = 0.4$
0.20	$131.3869i$	$129.001i$	$122.7276i$	$110.9159i$	$91.1455i$	$59.3517i$	$7.0564i$
0.15	$66.3475i$	$62.7713i$	$53.5830i$	$36.9280i$	$10.0682i$	–	–
0.10	$16.015i$	$10.3104i$	–	–	–	–	–
0.08	$0.9712i$	–	–	–	–	–	–

Table 3. Unstable QNMs for charged GB black brane perturbation of tensor type for fixed dimensionality ($D = 6$) and $k_3 = 500$.

with negative valued potential numerically and find some unstable QNMs (see tables 1, 2 and 3). Among these tables, we can find that the real part of ω is vanishing, while the imaginary part of ω is positive.

Table 1 demonstrates that the unstable modes of the black brane are suppressed as D increases. This confirms the result obtained in ref. [24]. From table 2 and 3, we see that lower value of charge (a) and λ stabilize the perturbation, while the lower value of D strengthens the instability. The reason for why higher D stabilize the perturbation is because that no matter how big D is, λ (i.e.$\lambda = (D-3)(D-4)\alpha'/l^2$) is bounded by $1/4$ which means that for fixed l, $\alpha' \to 0$ as D increases. Moreover, $\alpha' \to 0$ corresponds to vanishing Gauss-Bonnet correction in (2.1) and charged black branes in that regime are stable. For a complete numerical analysis of QNMs in Gauss-Bonnet theory, one may refer to [24] and references there in.

5 Conclusions and discussions

In summary, we have computed the dimensionality dependence of η/s for charged black branes with Gauss-Bonnet correction. The ratio of the shear viscosity to entropy density in D-dimensional space-time was found to be $\eta/s = \frac{1}{4\pi}\left(1 - \frac{2\lambda}{D-3}[(D-1) - (D-3)a]\right)$. When $D = 5$, we can recover the result found in [20]. It is worth noticing that for non-zero charge the viscosity can never approaches even in 5D case.

While in [35], it was always assumed that $\lambda \leq \frac{1}{4}$, in this paper we have shown explicitly that both causality and stability constrained the value of λ to be bound by $1/4$ in the limit $D \to \infty$, but for fixed D, these two constraints are different. It is interesting to notice that from different physical processes (causality and stability), we obtain the same bound. One may further check whether this is a coincidence or not.

The instability of charged black brane with Gauss-Bonnet correction was also analyzed in this paper. The result shows while higher value of charge (a) and λ strengthen the perturbation, the unstable modes of charged black brane are suppressed as D increases. The suppression of the unstable modes for higher D can be explained from the fact that λ is bounded by $1/4$ and $\alpha\,'$ approaches zero as D increases.

Acknowledgments

We would like to thank Y. Matsuo, F. W. Shu and T. Tsukioka for useful discussion at the early stage of this work. The work of XHG is partly supported by Shanghai Leading Academic Discipline Project (project number S30105). The work of SJS was supported by KOSEF Grant R01-2007-000-10214-0. This work is also supported by Korea Research Foundation Grant KRF-2007-314-C00052 and SRC Program of the KOSEF through the CQUeST with grant number R11-2005-021.

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